

Temperature and Heat.

What is temperature, T ? (in our perspective)

T quantifies physical properties of matter consists of numerous particles / molecules.

$T \longleftrightarrow$ state
characterised by macroscopic
quantities: Pressure, Volume, resistance ...

e.g. thermometer:

$T \longleftrightarrow$ Volume of liquid. $T \uparrow V \uparrow$.

Resistor in circuit

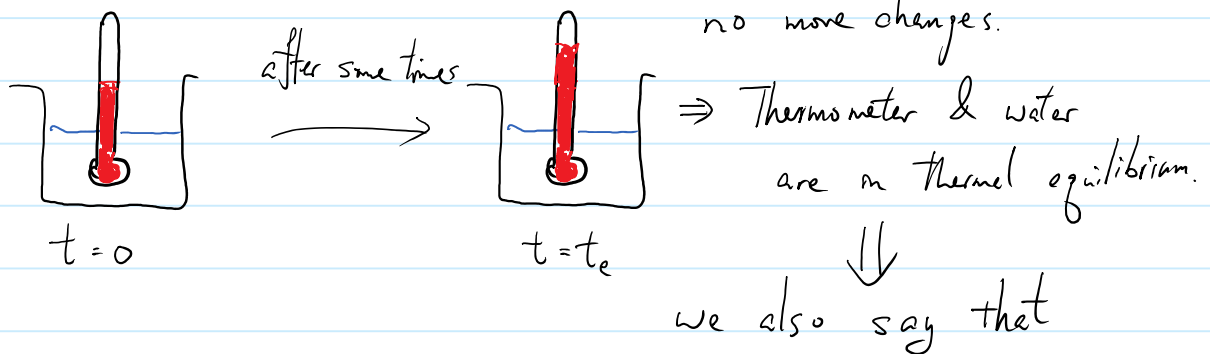
$T \longleftrightarrow$ resistance. $T \uparrow R \uparrow$

Thermal Equilibrium.

Thermal contact — energy transfer is allowed between two systems

When the states of two thermally contacting systems remain unchanged, the systems are called ^{being} in thermal equilibrium.

e.g. a thermometer in water



They have the same Temperature!

Thermal Equilibrium = same temperature.

Zero-th Law of Thermodynamics

If A & B are each in thermal equilibrium with C,
then A & B are in thermal equilibrium with each other.

Different units/scales of temperature.

Fahrenheit (T_F)	Celsius (T_C)	Kelvin (T)
• Freezing point of water 32°F	• Freezing point of water 0°C	• Lowest possible temp. 0 K
• Boiling point of water 212°F	• Boiling point of water 100°C	• Triple pt. of water $273.16\text{ K } (=0.01^\circ\text{C})$ @ $p = 0.006\text{ atm.}$
• human body temp. $\sim 96^\circ\text{F}$	• human body temp. $\sim 37.5^\circ\text{C}$	
set to fit human body temp.	set to fit water freezing & boiling pt.	most objective scale (more in Lecture 23-24)

Conversion:

$$T_F = \frac{9}{5} T_C + 32$$

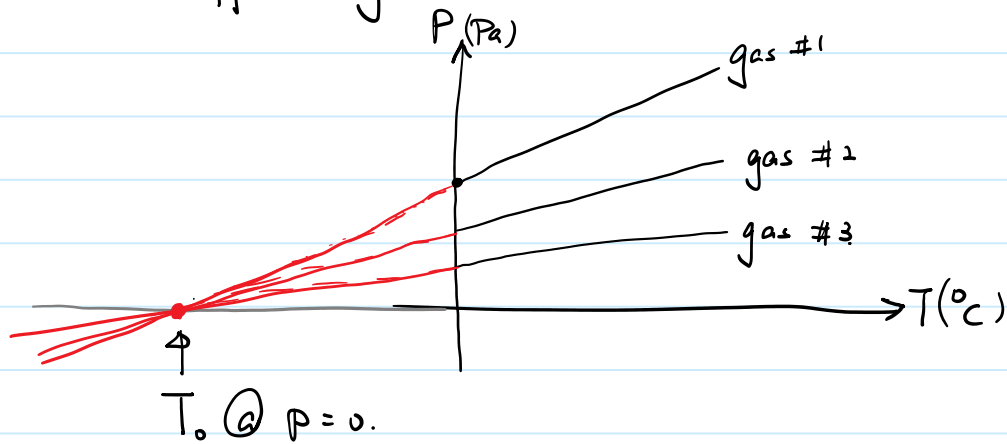
$$T_C = T - 273.15\text{ K}$$

Temperature and gas

Given a container filled with gas. at a fixed volume.

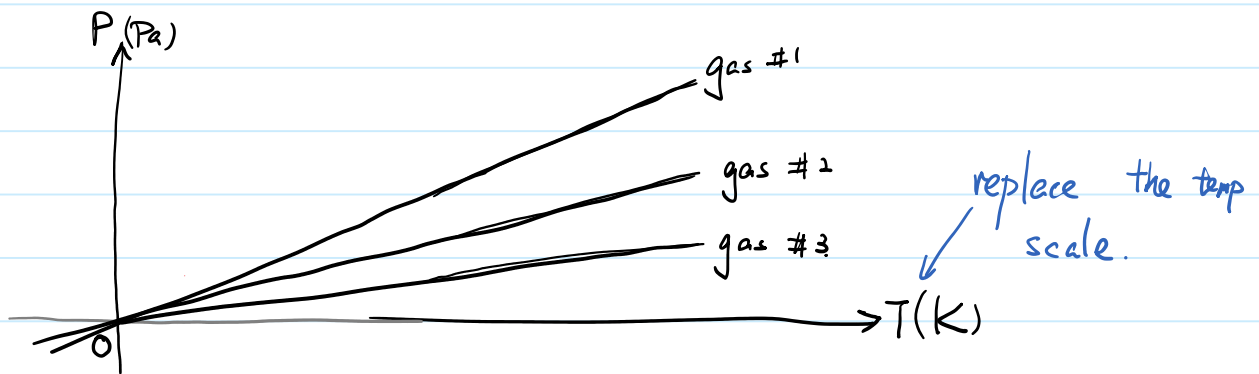
Temp. $\propto$ Pressure of the gas.

For different gases,



absolute zero. $T_0 = -273.15^{\circ}\text{C} = 0 \text{ K}$.

Plotting the diagram with the new T -axis in unit of Kelvin:



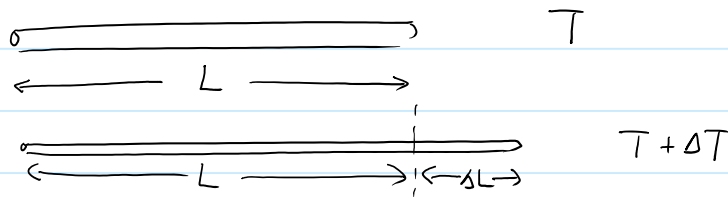
$$\frac{P}{T} = \text{constant} \quad \leftarrow \text{(value can be different for different gas)}$$

Kelvin Scale can be determined using water at fixed volume and its triple point as,

$$T = \frac{P}{P_3} T_3 = \frac{273.16 \text{ K}}{0.006 \text{ atm}} \cdot P$$

Thermal Expansion

Linear expansion



$$\text{fractional change} = \frac{\Delta L}{L} \propto \Delta T$$

$$\Rightarrow \frac{\Delta L}{L} = \alpha \Delta T \xrightarrow{\text{infinitesimal change}} \frac{dL}{L} = \alpha dT$$

↑
coefficient of linear expansion.

e.g. $\alpha_{\text{Aluminium}} = 2.4 \times 10^{-5} \text{ K}^{-1}$

$\alpha_{\text{glass}} = 0.4 \times 10^{-5} \text{ K}^{-1}$

open cold jar lid $\Rightarrow$ warm the lid.
lid expands more than glass.



Volume expansion.



$$\frac{\Delta V}{V} = \beta \Delta T \rightarrow \frac{dV}{V} = \beta dT$$

e.g. $V = L^3 \text{ (cube)} \Rightarrow \frac{dV}{V} = \frac{1}{L^3} 3L^2 dL = 3 \frac{dL}{L} = 3\alpha dT \equiv \beta dT$

$\underline{\beta = 3\alpha}$

Heat = energy transfer due to temperature difference. (instead of work)

Heat and temperature change

Heat $\Delta Q > 0$ if Heat enters to the system

$\Delta Q < 0$ if Heat exits from the system

for any object

$$\underline{\Delta Q = C_{\text{bulk}} \Delta T}$$

C_{bulk} = heat capacity of the object, which can be a mixture of different materials (bulk)

for single material

$$\underline{\Delta Q = m c \Delta T}, \quad c = \text{specific heat of the material}$$

e.g. water $c_w \sim 4200 \text{ J/kg}\cdot\text{K}$

for gas

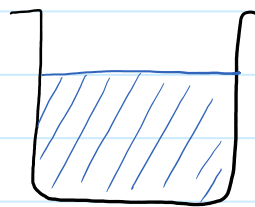
$$\underline{\Delta Q = n C_{\text{mol}} \Delta T}, \quad C_{\text{mol}} = \text{molar heat capacity}$$

Latent Heat - Amount of heat per one unit of mass

to change the substance from one phase to another.
e.g. gas, liquid, solid...

- During the phase change/transition, $\Delta T = 0$
e.g. vapourization, freezing...

Example.



Water

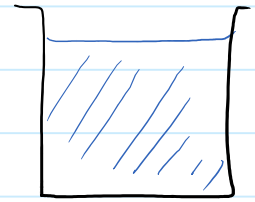
$$m_w = 0.25 \text{ kg}$$
$$T_w = 25^\circ \text{C}$$

+



Ice

$$m_I = ?$$
$$T_I = -20^\circ \text{C}$$



Water

$$T_f = 0^\circ \text{C}$$

Find m_I .

Assuming the process occurs in a thermal insulator.

$$\Delta Q_{\text{sys}} = 0$$

$$\Delta Q_{\text{water}} + \Delta Q_{\text{ICE}} = 0$$

$$\Delta Q_{\text{water}} = m_w c_w \Delta T = m_w c_w (T_f - T_w)$$

$$\Delta Q_{\text{ICE}} = \Delta Q_{\text{melting}} + \Delta Q_{T_I \rightarrow T_f}$$
$$= + m_I L_{\text{fusion}} + m_I c_I (T_f - T_I)$$

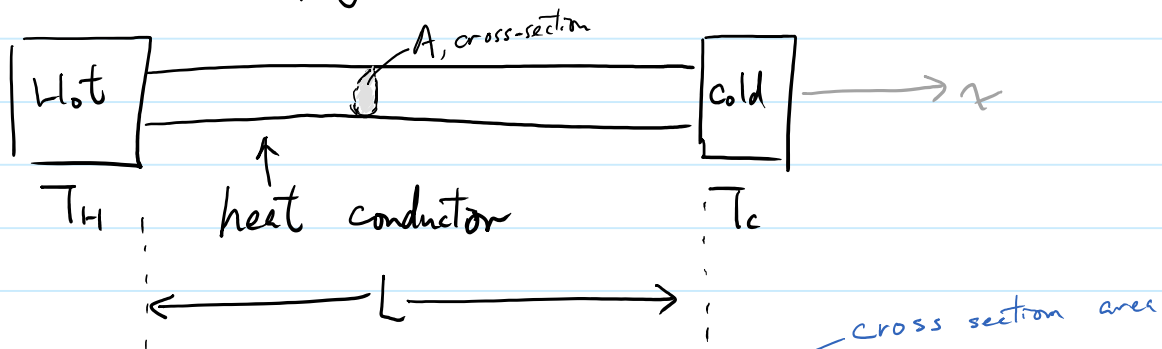
↑
Ice absorbs
energy in
melting

$$\Rightarrow m_I = \frac{m_w c_w (T_f - T_w)}{c_I (T_f - T_I) + L_{\text{fusion}}}$$

Mechanism of Heat Transfer

Conduction, Convection & Radiation

Conduction (physical contact.)



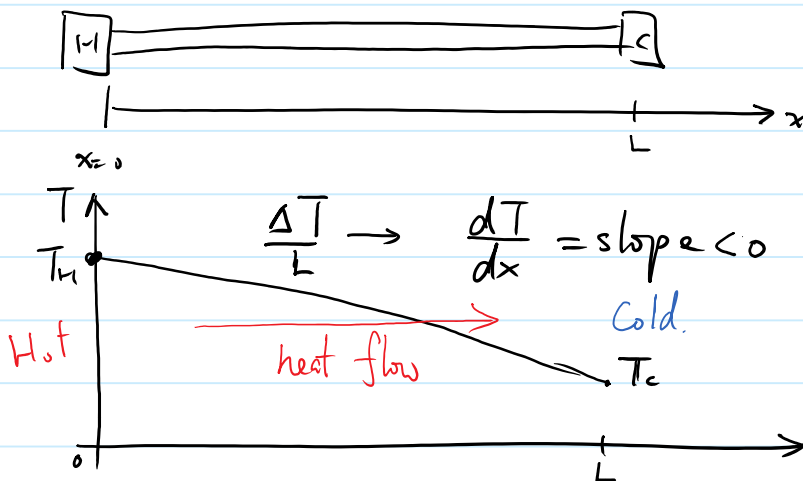
Heat flow per unit time = $H = \frac{k A (T_H - T_C)}{L}$
(unit: W)

Annotations: k is thermal conductivity (material dependence), $\frac{W}{mK}$. L is length of conductor.

Thermal conductivity (material dependence), $\frac{W}{mK}$.

$\frac{T_H - T_C}{L} = \frac{\Delta T}{L}$ = Temp. gradient, diff. in Temp. over distance

Consider temperature as a function of x .

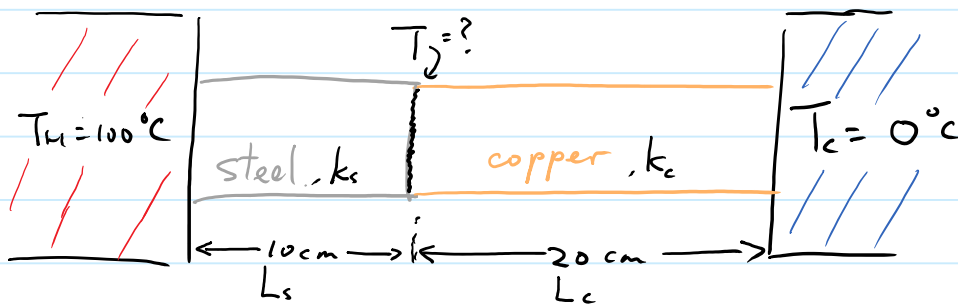


Since $\frac{dT}{dx}$ is opposite to the direction of the heat flow,

$H \propto -\frac{dT}{dx}$

Define $H > 0$ when heat flow to $+x$, $H = -kA \frac{dT}{dx}$

Example. conducting heat with two materials.



$$k_s = 50.2 \text{ W/mK} \quad , \quad k_c = 385.0 \text{ W/mK}$$

Find T .

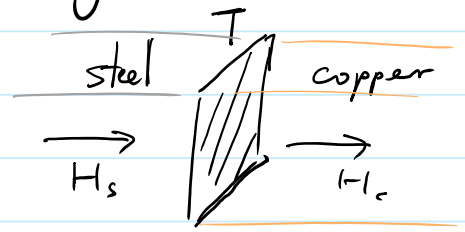
$$\text{Heat flow along the steel} = H_s = k_s A \frac{T_u - T}{L_s}$$

$$\text{Heat flow along the copper} = H_c = k_c A \frac{T - T_c}{L_c}$$

At steady state, $H_s = H_c$ otherwise there is net heat flowing in or away at the boundary.

$$\Rightarrow H_s = H_c$$

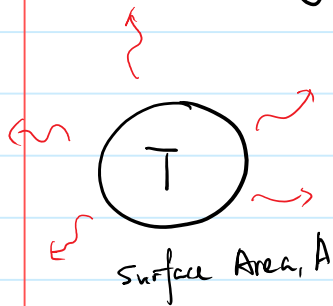
$$\Rightarrow \frac{k_s (T_u - T)}{L_s} = \frac{k_c (T - T_c)}{L_c}$$



$$\Rightarrow T = 20.7^\circ\text{C}$$

Radiation

Any body ($T > 0 \text{ K}$) radiates energy.



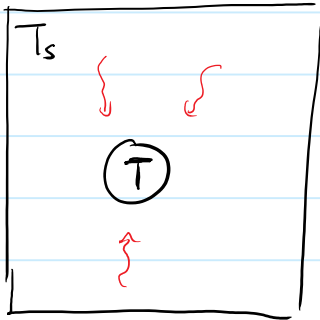
Radiation Power

$$H_{\text{rad}} = A e \sigma T^4 \quad [\text{W}]$$

$$\begin{aligned} \sigma &= \text{Stefan-Boltzmann Constant (universal)} \\ &= 5.6704 \times 10^{-8} \text{ W/m}^2/\text{K}^4 \end{aligned}$$

e = emissivity of object. $0 \leq e \leq 1$

A = Total surface area.



The environment surrounding (T_s) the object also radiates. The object will absorb the radiation according to.

$$H_{\text{abs}} = A e \sigma T_s^4$$

⇒ The net heat flow due to radiation is

$$H_{\text{net}} = H_{\text{rad}} - H_{\text{abs}} = A e \sigma (T^4 - T_s^4)$$

Human body at $T = 30^\circ\text{C}$ with surface area 1.2 m^2 and emissivity $e \approx 1$ surrounded by temperature 20°C .

$$\begin{aligned} H_{\text{net}} &= A e \sigma (T^4 - T_s^4) \\ &= (1.2)(1)(5.67 \times 10^{-8}) \cdot (303^4 - 293^4) \\ &= 72 \text{ W} \end{aligned}$$

i.e. the human needs to generate 72 W to keep the body temp.